Subalternation and Mathematical Physics

In the philosophy of science few notions are more basic than that of subalternation. It is, in fact, the pivotal concept in the whole problem of mathematical physics. And yet it is a notion to which modern Thomists have given only slight attention. One looks in vain in contemporary Thomistic literature for even an attempt toward an adequate analysis of its significance. Indeed, there has been so much loose and ambiguous handling of the term that the clear and sharp outlines it possessed in the minds of the earlier Thomists have to a large extent been lost. The purpose of this article is to restore these outlines, and to show the relevance of the concept for mathematical physics.

Subalternation is sometimes defined in terms of the application of one science to another, or the dependence of one science on another, or the subordination of one science to another. Its notion involves all of these things, but they do not adequately explain its proper meaning. In the first place, not every case of the application of one science to another is a case of subalternation. For example, in the philosophy of science there is a kind of application of metaphysics to experimental science. But this does not involve the subalternation of experimental science to metaphysics. The philosophy of science is a metaphysical study, in so far as it pertains to wisdom to make a critique of the nature of all the sciences including itself. Secondly, subalternation is not coterminous with dependence. For example, theology, in so far as it makes use of philosophy, may in some sense be said to be dependent upon it. But it is not subalternated to it[1]. Thirdly, the notion of subordination is not sufficient to explain the meaning of subalternation. For, philosophy is subordinated to theology, but it is not subalternated to it[2]. Moreover, all practical science is in some way subordinated to speculative science, but this subordination does not necessarily involve subalternation. It is true that some practical sciences, such as medicine, agriculture, etc., are subalternated to the science of nature, but that is because of the peculiar character of the relation that obtains between them, as we shall presently explain.

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1. Cf. JOHN OF ST. THOMAS, *Cursus theologicus* (ed. SOLESMES), t.1, pp.396 ff.

2. The term "subalternation" is sometimes incorrectly used by modern authors to explain the relation between philosophy and theology. Cf. E. GILSON, *L'Esprit de la philosophie médiévale*, Paris, Vrin, 1932, p.4: «Alors que le rationaliste pur place la philosophie au sommet et l'identifie à la sagesse, le néo-scolastique la subalterne à la théologie, qui reste seule à mériter pleinement le nom de sagesse; mais pourquoi certains néo-scolastiques pensent-ils que même subalternée à la théologie, leur philosophie demeure identique en nature à celle qui ne reconnaît aucune Sagesse au-dessus d'elle?»

One of the difficulties encountered in the problem of subalternation arises out of the fact that the term is used in a variety of ways. Perhaps the best means of arriving at the positive meaning of the term is by considering the different ways in which one science may be subalternated to another. John of St. Thomas distinguishes three types of subalternation[1]. One science may be subalternated to another either by reason of the end it pursues, or by reason of the principles it employs, or by reason of the subject it considers. Let us examine briefly each of these types.

I. THE KINDS OF SUBALTERNATION

Subalternation that derives from an end pursued is, as the very terms suggest, proper to the practical order; it is found in the practical sciences and in the arts. When the end of one science, though truly an end within its own order, is subordinated to the end of a higher science in such a way that it is controlled and directed by it, the first science is said to be subalternated to the second. Thus, for example, military science is subalternated to political science. It is important to note that the first end must be truly an end within a certain order, for if it is only a means, if the higher science uses it merely as an instrument, there is no real distinction of sciences, and hence no subalternation. In this first type we are dealing with subalternation in a very broad and improper sense. For subalternation implies the dependence of one science upon another with respect to the manifestation of truth, and very often when one science is subalternated to another by reason of its end there is no dependence of this kind, but rather dependence with respect to use, control, direction, and command, — something akin to what is found in the interrelation of the virtues, as, for example, in charity's command over temperance. And this follows from the very nature of the practical order, whose object is not the true as true, nor even the good as true, but the good as good. It is only in the speculative order that subalternation in the proper sense of the term is found, for the object of this order is always the true, and consequently subalternation in this order involves a manifestation of truth. We are particularly interested in the subalternation of the speculative sciences.

One speculative science may be subalternated to another in two ways: either by reason of its principles alone, or by reason of its subject. The first type of subalternation is found when a lower science borrows from a higher science the principles necessary to illuminate its "own" domain, and thus becomes dependent upon it. But in order to have subalternation of this kind in the full sense of the term, the dependence must be necessary and essential, that is to say, the lower science must be lacking in self-evident principles within its own domain, and thus be forced to reach up to a higher science to have its principles made evident. This type of dependence is found in the subalternation of supernatural theology to the

1. Cf. *Cursus philosophicus* (ed. REISER), t.1, pp.795ff.

science of the blessed. Theology does not resolve its demonstrations into principles that are self-evident to the theologian as such. These principles, accepted by faith in the wayfarer, have their intrinsic evidence in a higher science — the science of the blessed in heaven. It is in this higher science that the principles of theology, both of the wayfarer and of the comprehensor, find their manifestation and their proof. That is why theology is essentially subalternated to the science of the blessed, even in the blessed themselves.

It is extremely important to insist upon the difference between this kind of dependence and the kind of dependence that the philosophy of nature and the other sciences have upon metaphysics. It is true that in some sense all of the sciences receive their principles from metaphysics, for, as St. Thomas says, metaphysics "confers principles upon all the other sciences[1]." Nevertheless, the lower sciences do not depend upon metaphysics for the evidence of their principles. They are capable of resolving their demonstrations into self-evident principles which are proper to them. They do not have to turn to metaphysics to have the truth of their principles made manifest or proved. Metaphysics explains the principles of the other sciences and defends them by a reduction *ad impossibile*, but it does not prove them in an a priori fashion. The principles of the other sciences come under the influence of those of metaphysics only in the sense that metaphysics is the most universal and the most basic of all the sciences.

It is true that the other sciences may sometimes use metaphysical principles in their demonstrations. It is likewise true that they may sometimes employ principles taken from the science of logic. But this amounts to no more than an occasional borrowing from these other sciences; it merely means the use of an extrinsic proof. All this explains why the dependence of the other sciences upon metaphysics and logic is not subalternation in the full sense of the word. And if the term subalternation is applied to this kind of dependence it should be made very clear that it is only a question of subalternation in a very partial and limited sense[2].

Now for our purpose it is not subalternation by reason of the principles alone that is of particular interest, but subalternation by reason of the subject. In this third type we have subalternation in the most perfect sense of the word[3]. Let us try to see why this is so.

This third species of subalternation arises when the subject of one science falls under the subject of another science. But as St. Thomas points out in the twenty-fifth lesson of the first book of the *Posterior Analytics*, one subject may fall under another in two ways. First of all, it may merely be a question of a more specific subject being contained in a more generic

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1. *In Boetium de Trinitate*, q.6, a.1.

2. The earlier Thomists sometimes called this type of dependence subalternation *secundum quid*, but denied that it was subalternation *simpliciter*. Cf. J. OF ST. THOMAS, *Curs. phil.*, t.1, p.798a30.

3. *Ibid.*, p.796a10: "Tertius modus inducit propriissimam subalternationem."

subject, in the way in which, for example, animated mobile being falls under mobile being. Here, as is evident, there is no real distinction of science and hence no possibility of true subalternation. Every science explains its subject by division as well as by definition, and consequently in order to have the formal distinction of science that is required for subalternation, it is not sufficient that one subject add an essential, specific difference to the other. And this explains why many of the apparently mixed sciences, such as astrophysics, biochemistry, etc., do not involve true subalternation, since they arise merely out of the union of two branches of the same science[1]. There is, consequently, a world of difference between the mixed character of these sciences and that of mathematical physics in which physics is truly subalternated to mathematics.

Because the subalternated science must be properly extrinsic to the subalternating science, the difference which the subject of the one adds to the subject of the other must be extrinsic and accidental. An example will make this point clear. Let us take the geometrical notion of "line." We may add to this notion in two ways. First of all, we may add the proper differences "straight" and "curved," and thus arrive at two specific subjects "straight line" and "curved line," both of which fall under the generic subject, "line." By doing this we do not arrive at any new science, since the science which deals with a certain genus necessarily deals with all the proper species which fall under it. But it is also possible to add to the notion of line the extrinsic and accidental difference "visual," and thus arrive at a new subject, "visual line[2]." This new notion is not a proper species of the generic geometrical notion of line. Hence it does not fall under the science of geometry in the sense of being a part of its subject. In fact it makes for a new science, the science of optics, known to the ancients as *perspectiva*. This new science, while not falling under geometry in the sense of being a part of it, does come under it in some way, since the notion of line which is compounded with the notion of visual to constitute its subject is borrowed from geometry. In other words, optics is subalternated to geometry by reason of its subject.

Perhaps another simple example will clinch the point we are trying to make. We may add to the generic arithmetical notion of number the differences "odd" and "even," both of which pertain essentially to the subject of arithmetic. But we may also add to the notion of number the extrinsic and accidental notion of sound and thus arrive at a compound subject which constitutes a new science, distinct from arithmetic, but subalternated to it — the science which the ancients called *musica*.

Now subalternation by reason of the subject always involves at the same time subalternation by reason of the principles. This should be

1. We shall show elsewhere that according to the soundest Aristotelian an Thomistic tradition there can be no specific distinction between the different branches of natural science.

2. As we shall point out later, we must not confuse this "visual line" with the visual line that is a common sensible. In subalternation the abstract mathematical line is, as it were, superimposed upon the latter, thus constituting the "visual line" as we understand it here.

fairly evident from the examples just cited. Since the formal subject of the subalternated science is constituted by the addition of an accidental difference to the subject of the subalternating science, the subalternated science cannot treat its subject and prove its properties except by having recourse to the conclusions of the subalternating science. But subalternation by reason of the principles does not always involve subalternation by reason of the subject. The contrast between the way theology is subalternated to the science of the blessed and the way optics is subalternated to geometry brings this point out with sufficient clarity. As we saw, supernatural theology must turn to the science of the blessed in order to find the evidence of its principles. Nevertheless, its subject is not constituted by the addition of an accidental difference to the subject of the science of the blessed. It is, in fact, the very same subject viewed under two different lights: the light of virtual revelation on the one hand, and the light of vision on the other. But the difference between geometry and optics does not consist merely in two different ways of viewing the same subject. In the first instance, we have a simple notion that prescinds from all sensible matter. In the second, we have a compound subject made up of this simple notion plus an extrinsic and irreducible element which involves sensible matter. There is a world of difference between these two types of subalternation. In the first type, the subalternated science remains a simple science. In the second type, it becomes a complex science, a *scientia media*, because its formal subject is compounded of elements which involve two different levels of intelligibility.

The three fundamental types of subalternation just described are the only ones mentioned by John of St. Thomas in the article cited above. We may well wonder whether the list is exhaustive. For St. Thomas in his commentary on Boethius' *De Trinitate*[1] gives us a case of subalternation which does not seem to fall under any of the three groups listed by his disciple. We are referring to the case already mentioned earlier in this article in which the practical sciences of medicine, agriculture, etc. are subalternated to the speculative science of nature. We pointed out that this subalternation does not arise merely from the subordination that all practical science has to speculative science, but from the special character of the dependence which these few practical sciences have upon the speculative science of nature. St. Thomas brings out the nature of this special relation with great clarity and precision:

> Though the curable body is a natural body, it is not, however, the subject of the science of medicine in so far as it is curable by nature, but in so far as it is curable by art. Since, however, in the cure which art brings about, art ministers to nature (for some natural power effects the cure through the help of art), the reason or cause of the operation of art must derive from the properties of natural things. And thus the science of medicine is subalternated to physics, and for the same reason alchemy and the science of agriculture and all other sciences of the same kind. Hence it remains true that physics in itself and in all its parts is a speculative science, although some practical sciences are subalternated to it[2].

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1. Q.5, a.1, ad 5.
2. *Ibid.*

It does not seem possible to fit this type of subalternation directly into any of the three groups described above. It is not a case of subalternation by reason of the end, for we do not have one practical science subordinated to another practical science. For the same reason it is not a question of subalternation because of the principles, for a practical science cannot receive its "proper" principles from a speculative science. Finally, there is no possibility here of subalternation by reason of the subject, for elements from a practical science cannot be compounded with elements from a speculative science to constitute the subject of a simple, unified science. As a matter of fact John of St. Thomas, after explaining the three types of subalternation, explicitly denies that medicine is subalternated to natural science: "medicine deals with curable bodies, and yet it is not subalternated to philosophy, which deals with bodies[1]." From the context, however, it is evident that he is merely denying the possibility of subalternation by reason of the subject. And even though the way in which medicine and agriculture are subalternated to natural science does not fit directly into any of the three groups listed by John of St. Thomas, it may be reduced to the second group. For while it is true that a practical science cannot receive its principles from a speculative science, the principles of medicine and agriculture are completely determined by the principles of natural science because of the unique character of the relation existing between these sciences. The adage, "art imitates nature," applies here in a special way[2]. Ultimately the end of medicine and the end of nature are the same; the operations of the art imitate nature to further the healing operations of nature. The operations of this art would be performed by nature itself were it not so determined *ad unum*. And if performed by nature, they would, in turn, be similar to the operations of art, for both nature and art proceed *secundum vias determinatas*, according to ways specified by the end, which is in both cases the same[3].

1. *Curs. phil.*, t.1, p.796b40. In this context, pliilosophy means the science of nature.

2. "...Secundum quod Aristoteles in VII *Metaphysicae* docet, artium quaedam sunt in quarum materia non est aliquod principium agens ad effectum artis producendum, sicut patet in aedificativa: non enim est in lignis et lapidibus aliqua vis activa movens ad domus constitutionem, sed aptitudo passiva tantum. Aliqua vero est ars in cuius materia est aliquod activum principium movens ad producendum effectum artis, sicut patet in medicativa: nam in corpore infirmo est aliquod activum principium ad sanitatem. Et ideo effectum artis primi generis nunquam producit natura, sed semper fit ab arte: sicut domus omnis est ab arte. Effectus autem artis secundi generis fit et ab arte, et a natura sine arte: multi enim per operationem naturae, sine arte medicinae, sanantur. In his autem quae possunt fieri et arte et natura, *ars imitatur naturam:* si quis enim ex frigida causa infirmatur, natura eum calefaciendo sanat; unde et medicus, si eum curare debeat, calefaciendo sanat."—*Contra Gentes*, II, c.75. For further expositions of the adage, cf. *In Politicorum*, Prologus; *In II Physicorum*, lect.4, n.6.

3. "...Non deliberare contingit alicui agenti, non quia non agit propter finem, sed quia habet determinata media per quae agit. Unde et natura, quia habet determinata media per quae agit, propter hoc non deliberat. In nullo enim alio natura ab arte videtur differre, nisi quia natura est principium intrinsecum, et ars est principium extrinsecum. Si enim ars factiva navis esset intrinseca ligno, facta fuisset navis a natura, sicut modo fit ab arte. Et hoc maxime manifestum est in arte quae est in eo quod movetur, licet per accidens, sicut de medico qui medicatur se ipsum: huic arti enim maxime assimilatur natura. Unde patet quod natura nihil est aliud quam ratio cuiusdam artis, scilicet divinae, indita rebus, qua ipsae res moventur ad finem determinatum: sicut si artifex factor navis posset lignis tribuere, quod ex se ipsis moverentur ad navis formam inducendam."—*In II Phys.*, lect.14, n.8.

It would seem that if the concept of subalternation is conceived as embracing all of the various cases we have described, it can hardly have a strict unity. Nevertheless, there are two kinds of subalternation in which the concept is realized in its proper and strict sense, and in which it has a definite unity. We refer to subalternation by reason of the principles, in which there is an essential relation of dependence between the subalternated science and the subalternating science, that is to say, the former receives its "proper" principles from the latter, and to subalternation by reason of the subject. When the earlier Thomists speak of subalternation, it is usually this strict and proper sense of the concept that they have in mind, and it is in this sense that we shall speak of it hereafter.

And now, having reduced the notion to this definite meaning, we must undertake to explain in what its essence consists. But before pursuing this analysis, it is worth while pausing at this point to remark that every effort should be made to maintain a clear-cut distinction between the various kinds of subalternation we have been describing. As we pointed out above, this has not always been done by modern Thomists. We are being told by more than one contemporary writer, for example, that the philosophy of nature is a *scientia media*, born of a union of the first and third degrees of abstraction, or, even worse, arising out of the application of metaphysics to the data of empirical science. And we consider it quite misleading, unless all the necessary qualifications and distinctions are made, to insist, as some authors do, that in modern times mathematics has come to occupy the same position in relation to the experimental sciences that metaphysics held for the earlier Thomists.

II. THE NATURE OF SUBALTERNATION

The intrinsic nature of subalternation follows from the intrinsic nature of science itself. Science, in the strict Aristotelian sense of the term, is certain knowledge of things in their causes, and for the human intellect this means knowledge arrived at by a process of demonstration. Now knowledge that is arrived at by demonstration is never self-evident knowledge. Conclusions do not have their evidence from themselves, but from something else, namely from the immediately evident principles from which they have been derived. That is why the intellectual virtue of science is essentially dependent upon another intellectual virtue, known as the habitus of understanding (*intellectus principiorum*), which is the habitus that enables the mind to grasp immediately the truth of self-evident principles. Now the essential difference between a subalternated science and a science that is not subalternated is that the habitus of the latter is in immediate continuity with the habitus of understanding, whereas the habitus of the former

is only mediately in continuity with it, through the habitus of a higher science, known as the subalternating science[1].

In other words, no science is a science in and by itself, but in and by its continuity with a prior habitus, for without this continuity its conclusions cannot have the certitude that is necessary for scientific knowledge. A science that is not subalternated is a science that is in direct continuity with the habitus of understanding from which it immediately derives the evidence of its conclusions. On the other hand, a subalternated science is one that is in direct continuity with the habitus of a superior science, and only through this habitus is it in continuity with the habitus of understanding.

At this point it will be helpful to draw a contrast between the way supernatural theology is subalternated to the science of the blessed and the way other sciences are subalternated — not because we are here concerned with the subalternation of theology, but because the contrast will serve to accentuate the characteristic features that are found in the intermediary sciences in general and in mathematical physics in particular. In the subalternation found in all the other sciences besides theology, the proximate principles of the subalternated science are conclusions demonstrated by the subalternating science.

> ... A subalternated science does not employ the principles, but the conclusions of other sciences; for it borrows as its principles conclusions which are proved by a superior science, and does not, by going back to self-evident principles, employ the principles of the superior science[2].

When the subalternating science does not coexist in the same intellect with the subalternated science, these conclusions are taken on faith. But this does not mean that the principles of the subalternating science are taken on faith. For the intellect which possesses the subalternated science may possess the principles of the subalternating science by means of the habitus of understanding, without possessing the habitus of the subalternating science itself. In this connection John of St. Thomas writes:

> ...In the natural sciences it is impossible for the principles which are self-evident in the superior science by the habitus of understanding to be only taken on faith and not self-evident in the inferior science; for what is self-evident by the light of the habitus of understanding is self-evident to all; and the higher the principles are, and the higher the science to which they belong, the more evident are they to all because of their universality[3].

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1. ANNIBALDUS DE ANNIBALDIS (of whom Ptolemy de Luca wrote: "...Quem F. Thomas valde dilexit, facitque scripta super sententias ...quae nihil aliud sunt quam abbreviatio dictorum Fratris Thomae"), *In I Sententiarum*, d.1, q.1, a.1: "Primarum scientiarum proximum principium est intellectus, earum vero scientiarum, quae sua principia ab aliis supponunt, proximum principium est credulitas principiorum ab aliis suppositorum; primum vero earum principium est intellectus. Perficitur tamen certitudo istarum scientiarum cum per viam resolutionis in ipsum intellectum primorum principiorum perveniunt." Cf. P. MANDONNET, *Des écrits authentiques de saint Thomas d'Aquin*, Fribourg 1910, n.124, p.152.

2. J. OF ST. THOMAS, *Curs. theol.*, t.1, p.369b13.

3. *Ibid.*, p.364a30. In this passage the term "natural" is opposed to "supernatural."

This only refers, of course, to principles that are self-evident, and not to the postulates which a science may take as its principles. In this kind of subalternation there are two points to be noticed about the proper principles of the subalternated science: first, they are not evident; secondly, they are mediate, that is to say, they are the fruit of demonstration from principles that are evident. These two points are not identical, for it is possible for principles not to be evident without their being mediate. And in this distinction we find a fundamental difference between the kind of subalternation we have just been considering and the kind that is found in supernatural theology.

The proper principles of theology are not evident of themselves; but not all of them are mediate, since some are as first reasons, and others are truths consequent upon these reasons. Now as Cajetan points out[1], although both the element of lack of evidence and that of mediacy are ordinarily considered to pertain to the essence of subalternation in some way, the former pertains to it in a formal way, and the latter only in a material way. Hence, in order to have true subalternation it is not absolutely necessary that the proper principles of the subalternated science be conclusions; it is sufficient that they be not evident. In fact, John of St. Thomas maintains that in theology's use of principles that are not conclusions there is a fuller kind of subalternation than that found in the natural sciences where all the proper principles of the subalternated science are necessarily conclusions. For, whereas in the latter case, as we pointed out above, at least the principles from which the conclusions are drawn are evident, in the former the fundamental principles are in no way evident[2].

But here it is important to distinguish between two kinds of continuity, which for want of better terms we shall call objective and subjective. When the continuity is considered from the point of view of the science itself, prescinding from the scientist, it is objective; when it is considered from the point of view of the scientist it is subjective. Another way of expressing the same idea is to say that objective continuity is the continuity that a science has by its very essence, while subjective continuity is the continuity that it has because of its actual state. When a subalternated science is in its perfect state, there is subjective as well as objective continuity. But when it is in an imperfect state, subjective continuity may be lacking. And here it must be pointed out in passing that when Thomists raise the question about whether or not a certain subalternated science is in continuity with the subalternating science, it is to subjective continuity that they are referring, for, obviously there can be no question about objective continuity. But perhaps the best way to explain this distinction is by means of an example. The science of optics necessarily has objective continuity with the science of geometry, that is to say, its proximate principles are geometrical conclusions, which in turn have their evidence from their continuity with self-evident principles. But from the point of view

1. *In Iam*, q.1, a.2.
2. *Curs. theol.*, t.1, p.364a43.

of the student of optics this continuity may or may not exist. It exists if he is a mathematician as well as a student of optics. It does not exist if the geometrical conclusions which he applies to his particular matter are merely accepted by him on the authority of a mathematician without their intrinsic evidence being grasped.

In this distinction of the two kinds of continuity, we have the solution to a problem to which John of St. Thomas gives considerable attention[1]. The problem is this: when subjective continuity does not actually exist, is it possible for the subalternated science to be a true science? At first glance it would seem not. For scientific knowledge is necessarily certain knowledge. And how can knowledge be certain if it is reducible merely to principles which are held on authority and not to self-evident principles? Does not St. Thomas write: "... Whatever is known in strictly scientific fashion is known by relation to first principles which are self-evident to the intellect"[2]?

As we have just said, the correct solution of this problem lies in the distinction between subjective and objective continuity. Even when subjective continuity is lacking, objective continuity is always there, and that is sufficient to insure the truly scientific character of the subalternated science. For objective continuity means that the proper principles of the subalternated sciences are *de facto* demonstrated in the subalternating science, and thus there is the essential connection between the subalternated science and self-evident principles which St. Thomas demands in the text just cited.

This problem has particular significance for the science of theology, which, in this life, is based completely on faith. But it also has relevance for the question in which we are especially interested. For we can imagine the hypothetical case of a student of nature who, though unacquainted with the pertinent mathematical demonstrations that are presupposed, might accept the mathematical conclusions he needs on authority and employ them in his interpretation of natural phenomena. The conclusions concerning nature that he would be able to arrive at by using the borrowed mathematical conclusions as principles would express objective truth, even though they could not be called scientific truths from the point of view of the student himself.

From this we may conclude that a subalternated science is specifically the same scientific habitus whether there is subjective continuity with the subalternating science or not. For even when subjective continuity is lacking, the objective continuity establishes an essential relation between the subalternated and the subalternating sciences. It is this essential relation that determines the nature of the subalternated habitus. And this essential relation demands completion by subjective continuity. Hence, as long as subjective continuity is lacking, the habitus of the sub-

1. *Curs. phil.*, t.1, pp.799ff.
2. *Q. D. de Veritate*, q.14, a.9.

alternated science is in an imperfect state. But when it is acquired, no new habitus is born; the old habitus is merely brought to fullness and perfection. In this connection St. Thomas writes:

> ...No one who possesses a subalternated science achieves truly scientific knowledge except in so far as his knowledge is in some way in continuity with the knowledge of one who possesses the subalternating science. Nevertheless, the former is not said to have scientific knowledge of the things which he supposes, but of the conclusions which follow of necessity from the supposed principles[1].

At this point we must turn our attention to a highly significant passage of John of St. Thomas:

> ...Subalternation in the strict sense of the word does not arise from a mere borrowing of a principle from other sciences in order to employ it as an extrinsic and substitutional principle. The reason is that subalternation in the proper and strict sense of the word demands that a science be incapable by means of its own proper and intrinsic principles to reduce its demonstrations to self-evident principles, and be obliged to obtain evidence for its own principles by having recourse to another science which provides such evidence. If, however, it employs the principles of other sciences merely as extrinsic and substitutional, and if the recourse which it has through them to another science is merely for the sake of the evidence they provide, it is not intrinsically subalternated, for in so far as its own proper and intrinsic principles are concerned, it does not derive evidence from the other science, but only with respect to extrinsic principles. And it is in this way that we may determine whether or not subalternation is proper and intrinsic, namely by seeing whether it is found in the intrinsic and proper principles of the science, or merely in the extrinsic and borrowed principles[2].

These lines have two obvious references. In the first place, they refer to a point made by John of St. Thomas in the *Cursus philosophicus* which we have discussed earlier in this article: an occasional and extrinsic borrowing of principles from other sciences, such as metaphysics and logic, does not constitute subalternation in the strict sense of the word. In the second place, they refer to the immediate context in which the author shows that theology cannot be subalternated to philosophy even though it uses philosophical principles in its demonstrations, for first of all it does not take them as its own proper principles, and secondly it uses them only after having judged them in the light of its own wisdom by which they are approved as apt and fitting, and by which they acquire a certitude greater than purely natural certitude, so that the whole essence of the demonstration rests formally and ultimately upon the supernatural principle[3].

But it is not particularly because of these immediate references that we have introduced this passage here. Rather it is because some of the statements in it give rise to a problem which touches the very essence of the type of subalternation found in mathematical physics.

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1. *De Ver.*, q.14, a.9, ad 3.
2. *Curs. theol.*, t.1, p.369a39-b18.
3. To the remark that the use of philosophy in theology is like mixing wine with water, St. Thomas replies: "...Illi qui utuntur philosophicis documentis in sacra Scriptura redigendo in obsequium fidei, non miscent aquam vino, sed convertunt aquam in vinum." — *De Trin.*, q.2, a.3, ad 5.

As this passage of John of St. Thomas suggests, the earlier Thomists do not seem to have considered the case of what we shall call dialectical subalternation, that is to say, a type of subalternation in which the subalternating science does not give to the subalternated science in an intrinsic and adequate way the evidence of the principles that are proper to the subalternated science — one in which there is not realized a sufficiently perfect continuity between the two disciplines in question to permit the formation of a science in the strict sense of the term. Now this is the type of subalternation that is actually found in mathematical physics. And that is why we must develop this point a little further.

The earlier Thomists recognized the existence of mathematical physics, and they accurately analyzed its nature as an intermediary discipline that involves the fullest kind of subalternation — subalternation by reason of the subject. They carefully distinguished this type of subalternation from that found in theology where the principles alone are involved. Nevertheless, for them there was a fundamental parity between these two types of subalternation. Just as there was a perfect continuity between the principles of theology and those of the science of the blessed, so there was a perfect continuity between the principles of physics and those of mathematics — at least sufficiently perfect to permit mathematical demonstrations to be applied adequately to physical phenomena. It would seem that for Aristotle and the earlier Thomists the combination between the mathematical and the physical elements in the subject of mathematical physics was in a sense more intimate than it is possible to admit today. Because of a lack of refinement in their means of observation, they seem to have held that there are quantitative determinations in nature which come sufficiently close to the absolute state of perfection that they enjoy in the abstract mathematical world to allow for a true scientific handling of them in terms of mathematics. The heavenly bodies, for example, were for them perfect spheres, and consequently there was sufficient conformity between them and mathematical spheres to allow the mathematical properties of sphericity to be applied to them directly and adequately[1].

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1. Yet we must call attention to a very important passage from St. Thomas' commentary in I *De Caelo*, where he points out in admirably critical fashion that the whole Greek and Medieval theory of the celestial bodies can be no more than a plausible hypothesis. "Secundum signum ponit [Aristoteles, 270b12] ibi: *Accidit autem hoc et per sensum* etc.: quod quidem accipitur ab experientia longi temporis. Et dicit quod id quod probatum est per rationem et per communem opinionem, *accidit*, idest consequitur, sufficienter; non quidem simpliciter, sed sicut potest dici per comparationem *ad humanam fidem*, idest quantum homines possunt testificari de his quae parvo tempore et a remotis viderunt. Secundum enim memoriam quam sibi invicem tradiderunt astrologi, dispositiones et motus caelestium corporum observantes, in toto praeterito tempore non videtur aliquid transmutatum esse neque secundum totum caelum, neque secundum aliquam propriam partem ejus. Quod quidem non esset si caelum generabile et corruptibile esset: quaecumque enim generantur et corrumpuntur, paulatim et successive ad perfectum statum perveniunt, et ex eo paulatim recedunt: quod quidem non posset tanto tempore latere in caelo, si naturaliter generationi et corruptioni subjaceret.—Nec tamen hoc est necessarium, sed probabile. Quanto enim aliquid est diuturnius, tanto maius tempus requiritur ad hoc quod ejus mutatio deprehendatur; sicut transmutatio hominis non deprehenditur in duobus vel tribus annis, in quibus deprehenditur transmutatio canis, vel alicujus alterius animalis breviorem vitam habentis. Posset igitur aliquis dicere quod, etsi caelum sit naturaliter corruptibile, est tamen tam diuturnum, quod totum tempus

This does not mean, of course, that mathematical entities were realized as such in the physical universe, for that would involve a confusion of mathematics and physics, and Aristotle and St. Thomas go to great lengths in inveighing against those who proposed such a confusion[1]. But it does mean that some physical entities possessed a determination which was in close enough conformity with the perfect determination of mathematical entities for mathematics to give an adequate explanation of them. That is why Aristotle and St. Thomas could look upon the combination of mathematics and physics as giving rise to a science in the strict sense of the term.

It would seem that this particular application of their doctrine is open to modification. Because of our more highly refined instruments of research, we are no longer inclined to believe that such a verifiable conformity exists between physical and mathematical entities. The reason why they held this view was that they were without refined experimental instruments and had to depend upon rudimental sense experience. Now rough sense experience is extremely illusive. It often gives the impression that things in nature have a perfection which as a matter of fact they lack. The sense of touch may convey the notion that a surface is perfectly continuous and flat; the sense of sight may give the impression that a physical sphere is a perfect sphere. Consequently, when there is nothing else to go on but this rough experience, one is easily led to feel justified in positing the hypothesis that physical lines and figures reasonably and sufficiently approach mathematical exactness[2].

The refinement of our theories and instruments has emphasized the gap between physical and mathematical entities. We are only beginning to realize the practical implications of a principle that has long been recognized: all of our measurements are mere approximations[3]. Indeed it seems necessary to hold that mathematical physics, proceeding from positions which involve a tentative parallelization, can never produce science in the strictest sense of the term, that is, as Aristotle defines it in the *Posterior Analytics*[4]. However much the entities and functions of nature itself may approach mathematical precision, we have no adequate means at our disposal to reach such exactness. The ideal of mathematical physics seems to lie beyond history, and perhaps only a separated intelligence who is not a part of the physical world, and does not have to probe it with physical instruments could have rigorous scientific knowledge of this field. It is true that such positions and inferences may be provisionally confirmed and may prove very fruitful in practice. Yet the most we can

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cujus memoria potest haberi, non sufficit ad deprehendendam ejus transmutationem." —Lect.7, n.6. See also the *Praefatio* to the LEONINE edition of this work, particularly paragraph V, pp.x-xii.

1. Cf. *Metaphysics*, III and XIII.

2. See, however, *In XII Metaphysicorum*, lect.9 (ed. CATHALA), n.2565: *In I de Caelo*, lect.7 (ed. LEON.), n.6.

3. *Metaph.*, X, chap.1, 1053a1-25; S. THOMAS, *In X Metaph.*, lect.2.

4. I, chap.1.

say is that by constantly undergoing radical revision and renewal they move in the general direction of the truth. While it is exact to say that they do converge toward the truth, of no given theory can it be said that it achieves the proper reason of things even in its own sphere and that as a consequence it is necessarily true. To say that mathematical physics remains in a tentative stage with respect to proper reasons means that in this regard it proceeds after the manner of dialectics—not in the Hegelian or Marxist sense, but in the original and traditional understanding of dialectical position and inference. We shall go into this more thoroughly on another occasion. For the present it is enough to point out that science in the strict sense is not essential to true subalternation.

What has just been said about the opinion of Aristotle and the earlier Thomists might possibly give rise to a problem. For if they believed that there exist in nature entities and functions whose exactness comes reasonably close to that of mathematics, why are not these entities and functions within direct reach of a purely physical science? Why must they be approached through a *scientia media*—through a science that is at once something more and something less than pure physics? The answer is that even if the conformity between physical and mathematical entities were perfect, physics would still have to be subalternated to mathematics. For the concrete quantitative determinations of nature, in so far as they are natural, and inseparable from sensible qualities, are not susceptible of the free and rigorous elaboration allowed by mathematical quantity. For quantity is by its very nature more abstract than sensible qualities, and has reasons prior to those of the sensible qualities. This alone, it seems, would lead to subalternation.

A few general remarks remain to be made in order to complete our consideration of the nature of subalternation. In the first place, it should be evident from what has already been said that it is always a lower science which is subalternated to a higher science and not vice versa.

> . . . The more a science is concerned with things that have a greater degree of abstraction and simplicity, the more are its principles applicable to the other sciences; hence the principles of mathematics are applicable to the natural sciences but not vice versa, since physics presupposes mathematics, while the converse is not true[1].

A higher science may at times use the principles of a lower science, but then the dependence is only material and not formal, for the higher science in that case interprets the principles of the lower in terms of its own superior light[2]. In the *Posterior Analytics*, St. Thomas gives us an example in which a mathematical proposition is demonstrated in physics:

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1. *De Trin.*, q.5, a.3, ad 6.

2. Cf. Vassily Pavlov, *Mathematics for the Doctor in the Million*, in *Philosophy of Science*, Vol.II, n.1, p.48: "... An effort has been made ... to a pretense of applying the concrete sciences to the abstract ones. It has gone to the extent of naming new hybrids in inverse order as *physical mathematics*, (compare Einstein's 'physical geometry') ... *biological mathematics*, and the like... To this writer it still looks like the application of mathematics to biology rather than the reverse."

There are some propositions which cannot be proved except by the principles of another science; as a consequence it is necessary that they be supposed in the one science, while proved by the principles of another. Thus, that a straight line can be drawn from one point to another, is supposed by geometry, but proved by natural science, which shows that there is a line between any two points[1].

It should also be evident that the subalternated science and the subalternating science can coexist in the same intellect. In fact, this coexistence is the normal case, for it is synonymous with the subjective continuity we spoke of. One could not get very far in analytical geometry without possessing arithmetic and algebra, nor in mathematical physics without a personal knowledge of mathematics. In theology this coexistence or subjective continuity with the subalternating science is impossible in this life, but it will be realized in the next, for after death the habitus of theology will perdure, even though faith has disappeared.

Because of the close objective continuity between the subalternated and subalternating sciences a further question arises: do these sciences have the same object and the same subject? An adequate answer to this question demands several distinctions which, for the sake of convenience, we shall consider in connection with a general division given by John of St. Thomas[2]. It is to be understood, of course, that in speaking of the object or the subject of a science, we are using the term "science" to mean knowledge acquired by inference, as opposed to immediate knowledge of fact or of principle.

Now by the "object" of a science we mean whatever is known by way of inference, namely the conclusion, in which something is predicated of a subject for a reason contained in the premises. When we consider the inferred proposition or conclusion formally as "that which" is known, it is called the "material object" of the science. The "formal object", on the other hand, is that "by means of which" the material object or scientific conclusion is reached, namely the very reason for what is stated in the conclusion. This reason is contained in the principles or premises, and consists in a definition of a certain type, or at least something which is better known to us and which thus takes the place of a definition[3].

The "subject" of a science is that about which something is inferred and predicated in the conclusion. While the "material subject" is simply that about which the inference is made, the "formal subject" is the precise aspect under which the material subject is considered, e.g. corporeal things *qua* mobile (physical science), *qua* measurable (mathematical physics), *qua* being (metaphysics).

With these distinctions in mind, let us return to our question. Obviously the subalternated science may have the same material subject as the subalternating science. It may also have the same formal subject, but then there will be subalternation by reason of the principles only. Thus,

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1. I, lect.5, n.7.
2. *Curs. theol.*, t.1, p.402. For further distinctions, *Curs. phil.*, t.1, pp.818ff.
3. *In VI Metaph.*, lect.1.

theology and the science of the blessed have the same formal subject —*ratio deitatis*, not, however, the same formal object, for that would make them identical. This example brings out clearly the difference between the two kinds of subalternation, for, in mathematical physics, not only does the formal object differ from that of mathematics whose definitions prescind from sensible matter, but the formal subject differs as well, arising as it does from the addition of an extrinsic, accidental difference to the subject of the subalternating science. To understand what this involves, we must now analyze more closely the particular kind of subalternation found in the intermediary sciences.

III. SUBALTERNATION AND "SCIENTIA MEDIA"

Let us begin our analysis by considering the conditions required for an intermediary science, some of which have already been touched upon. In the first place, the subject of the subalternated science must contract the subject of the subalternating science by adding something to it. This addition cannot be a property flowing essentially from the subject of the subalternating science, since the same science deals with all the essential properties of its formal subject. Consequently, the addition must be an accidental difference rendering the subject extrinsic to that of the subalternating science. Nor will any kind of accidental difference be sufficient to constitute an intermediary science. For, not all accidental differences are the source of new scientific properties. For example, nothing fruitful results from the addition of "hot" or "cold" to the mathematical notion of "line;" properties of natural phenomena, as yet unknown may, however, be revealed by the addition of "visual," as the science of optics attests. In the same way, the addition of "visual" to the notion of number does not give rise to special scientific properties, while the addition of "sound" does, as is evident in the science of music.

It is important to understand accurately the accidental character of this difference we add to the subject of the subalternating science. It must not be understood to imply a mere accidental difference between the sciences themselves. We could hardly speak of distinct species of science, were the subject of the subalternated science but an accidental aggregate Indeed the latter discipline could actually be no more than mere sophistry. If this subject is to reveal new and constant properties of natural things, it must possess a *per se* unity. This is already clear from the mere fact that not every addition is useful, as we have just pointed out.

St. Thomas points out that the subject of the subalternated science stands in relation to the subject of the subalternating science as "material" to "formal[1]." The "formal" is contracted by application to the "material," as for example "line" by application to "visual." But the

1. *In I Posteriorum Analyticorum*, lect.25, n.2. *De Trin.*, q.5, a.3, ad 7.

two subjects do not merge to make up an *unum per se*, for that would make the latter a mere division of the former. They are, however, *per se* related. As John of St. Thomas carefully points out, the subject of the subalternated science is not the aggregate of the subject of the subalternating science and of the accidental difference, *sed respicit unum illorum per se, non tamen absolute, sed ut modificatum et connotatum per aliud*[1],—only one subject is considered *per se*, not, however, apart from the other, but as modified and connoted by the other. For example, the science of optics has as its direct subject a *per se* sensible line. It does not, however, consider this subject absolutely, but as modified and connoted, in our view of it, by the abstract mathematical line. Actually two quite different lines are held in view, though not in the same way. Given this relatedness, the results of the application of one to the other testify to its fecundity.

Obviously this peculiar relatedness has a foundation in nature. The mathematical line was originally separated by the mind from the common sensible "line" (whatever may be our success in attempting to reconstruct it without the slightest reference to experience). It is the geometrical line, purified by the abstraction proper to mathematics, that now serves to manifest the natural line which, as natural, is inseparable from sensible matter. Experience proves that this geometrical manifestation and illumination obtains rich results in the study of nature.

While the union of the mathematically abstract and the strictly natural can never make up a *per se unum*, the extrinsic modification and connotation may reveal rules to which nature conforms or, at least, approaches. The knowledge thus provided does not derive from the accidental aggregate as such, but from the subject as modified and connoted by the subject of the superior science. Perhaps an analogy may help to clarify this point. Paternity is something accidental to man in the sense that not all men are necessarily fathers. Nevertheless, a number of essential properties flow from the notion of man when it is considered precisely as connoting the notion of paternity, which do not arise when it is considered independently of this determination.

Now since the subject of a mixed science implies different levels of intelligibility, the question arises whether the abstraction employed in it is dual, or specifically one. John of St. Thomas explains that it is only one, and that it is a special intermediary abstraction which stands in between the two levels of intelligibility from which the elements have been borrowed, and which participates in the nature of both.

With regard to what is said about music and the other subalternated sciences, the answer is that in them the abstraction is not dual but one, in so far as the principles of the superior science by their application to a certain matter are rendered less abstract and consequently of a different scientific species. And that abstraction which they take on in a certain matter is one, and consequently they share in the nature of both sciences concerned — by means of one abstraction however — just as a mean, while remaining one, is said to share in the extremes[2].

1. *Curs. phil.*, t.1, p.797b21.
2. *Ibid.*, p.827a44.

The significance of the Thomistic doctrine of *scientia media* has not always been correctly understood. Thus, for example, Professor Salman writes:

> As for the *scientiae mediae*, whose theoretical importance has been greatly exaggerated, it is necessary to consider them as nothing more than a simple historical accident. A few of the easier problems had received at the hands of the Greek geometricians very precise solutions, and their mathematical character was as a result brought into clearer evidence. The belief arose, as a consequence, that the theory of vibrant cords, catoptrics, and astronomy were something distinct from the other less developed parts of physics. The difference, however, was only apparent, as we have pointed out above in bringing out the mathematical elements implicit in the rudimentary formulas of ordinary language. It is to be noted, moreover, that historically these intermediary sciences never entered directly into the classification of the sciences, but are added on in the answers to objections. They do not, as a matter of fact, derive normally from the theory of the degrees of abstraction, but are embarrassing factual data which the theorist must incorporate as best he can into a system which did not forsee them[1].

We fail to see any foundation for the objection that the intermediary sciences do not enter directly into the classification of the sciences. By the very fact that they are intermediary, a status based on simple fact and actual results, we could hardly expect them to fit directly any one of the three general types of knowledge which derive from the degrees of abstraction. If this is what Professor Salman has in mind when he says that they do not derive normally from the theory of the degrees of abstraction, his observation is perfectly true. But then it is an observation which calls for an explanation and perhaps for a restatement of how the general doctrine of abstraction is to be understood. On the other hand, there is a sense in which it must be said that they derive essentially from the degrees of abstraction. For it is only by seeing these sciences precisely as intermediary sciences, that is, as combinations of two different levels of intelligibility which arise out of two distinct kinds of abstraction, that we can understand their true nature. It is utterly impossible to grasp their meaning except in relation to the degrees of abstraction. That is why it is completely false to say that they are mere factual data which the philosopher must force arbitrarily into a synthesis that has no natural place for them. Nor did Aristotle or any of his great commentators ever show signs of embarrassment in this matter.

We feel that perhaps enough has already been said to show that the intermediary sciences were far from being "a simple historical accident," and that the difference between them and pure natural science is essential and not merely apparent. A further analysis would clarify and confirm this. Mathematical physics is specifically distinct from pure natural science because its very subject requires modification and connotation by the subject of mathematics, and because it must borrow from mathematics to establish its own principles. And yet the introduction of this extrinsic element into experimental physics is necessary and not arbitrary. The ancients recognized clearly both these points.

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1. *La conception scolastique de la physique*, in *Revue néo-scolastique de philosophie*, 1936, Vol.39, n.1, pp.38-39.

As for the remark that the theoretical importance of the intermediary sciences has been greatly exaggerated—we feel that the contrary is the case. The great epistemological implications latent in this doctrine and its relevance for modern physics have scarcely been recognized. We shall have occasion to show this by discussing the problem of subalternation as Aristotle does in connection with the two types of demonstration—*quia* and *propter quid*.

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